

SM3 10.1: Arithmetic Sequences

A sequence is an ordered list and a function. Sequences are useful for studying patterns in mathematics. Sequences are usually named with a single lower-case letter and are delimited by commas. Sequences can have any number of terms, which means they can be either finite or infinite.

We refer to individual terms of a sequence by using the name of the sequence with a subscripted index. So, a_4 represents the 4th term of a sequence called a .

Example: $p = 7, \pi, 100, 0.25, 11$
 p is a finite sequence because it has 5 terms.

Example: $q = 2, 5, 8, \dots$
 q is an infinite sequence because it has infinitely many terms.

$$p_1 = 7, p_2 = \pi, p_3 = 100, p_4 = 0.25, p_5 = 11$$

$$q_1 = 2, q_2 = 5, q_3 = 8, \dots$$

Mathematicians use ellipses (...) to mean "and so forth". Ellipses can be used at any location in a sequence to continue the sequence. When present at the edge of a sequence, ellipses indicate that the sequence is infinite in the direction of the ellipses.

A sequence is considered **arithmetic** when a common difference exists from term to term. We reserve lower-case d to mean the common difference of a sequence. You can add d to get from any term to the next term or subtract d to get from any term to the previous term. We call this fact the recursive rule, which states that if you want to find any term in an arithmetic sequence, you can just add the common difference to the previous term.

$$\text{Recursive Rule: } a_n = a_{n-1} + d$$

Example: $v = 157, 161, 165, 169, \dots$ Use the recursive rule to find v_7 .

We can only use the recursive rule if the sequence is arithmetic, which means it has a common difference. The common difference of sequence v is _____.

Use the recursive rule by adding the common difference to any term to find the next term. Keep going until you get the value you're looking for.

$$\begin{aligned} v_4 &= 169 \\ v_5 &= \quad + \quad = \\ v_6 &= \quad + \quad = \\ v_7 &= \quad + \quad = \end{aligned}$$

Given
Use the recursive rule with $n = 5$
Use the recursive rule with $n = 6$
Use the recursive rule with $n = 7$

Students typically do not find using the recursive rule to be challenging because the recursive rule feels like common sense. Unfortunately, it can be tedious. Imagine if we wanted to find $v_{123823567}$.

Often, it is too much work to find a term recursively. Let's discover a general solution to finding the n^{th} term of a sequence.

Find v_{123} of $v = 157, 161, 165, 169, \dots$

Sequence v is built by adding 4 to get to the next term.

To get to the second term, we added 4 to the first term.

To get to the third term, we added $4 + 4$ (that is 2 copies of 4) to the first term.

To get to the fourth term, we added $4 + 4 + 4$ (that is 3 copies of 4) to the first term.

⋮

To get to the 123rd term, we will add _____ copies of 4 to the first term.

⋮

In general, to get to the n^{th} term, we will add _____ copies of 4 to the first term.

$$\text{General Rule: } a_n = a_1 + (n - 1)d$$

$$v_{123} = 157 + (\quad)4 =$$

While the numbers involved with the calculation were large, at least we it was just one calculation!

Should you use the recursive rule or general for finding terms in a sequence?

HW10.1

Let $v = \frac{1}{2}, 9, 32, \ln 17, 4, e^3$. Evaluate the following:

1) Is v arithmetic; if so, find the common difference, d .

2) $v_1 =$

3) $v_3 =$

4) $v_4 =$

Let $a = 2, 7, 12, 17, 22, 27, 32, 37$. Evaluate the following:

5) Is a arithmetic; if so, find the common difference, d .

6) $a_1 =$

7) $a_3 =$

8) $a_5 =$

9) $a_9 =$

Let $b = 20, 13, 6, -1, -8, \dots$. Evaluate the following:

10) Is b arithmetic; if so, find the common difference, d .

11) $b_1 =$

12) $b_6 =$

13) $b_{10} =$

14) $b_{415} =$

Let $c = 0.3, 1.1, 1.9, \dots, 80.3, \dots$. Evaluate the following:

15) Is c arithmetic; if so, find the common difference, d .

16) $c_1 =$

17) $c_5 =$

18) $c_n = 80.3; n =$

19) $c_{29} =$

Let $q = \dots, 2000, 2004, 2008, \dots$. Evaluate the following:

15) Is q arithmetic; if so, find the common difference, d .

16) $q_1 =$

17) Describe a real world context for what q could represent.

